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Charm Mixing and CP Violation in the Standard Model *

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ABSTRACT

The Standard Model predictions for D^0-$\bar{D}^0$ mixing and CP violation in D decays are revised. The emphasis is put on obtaining the order of magnitude of the effects. In the case of mixing, the different approaches to the long distance contributions are carefully discussed. The size of CP asymmetries is discussed in general and some specific calculations are reviewed. The possibility of using kinematic signals is briefly described.

Charm mixing and CP violation are usually thought to be negligibly small in the Standard Model (SM) when compared to the same effects in the K and B systems. The question of how small is small becomes critical when we consider the possibility of high sensitivity charm experiments which could produce 10^8 reconstructed D mesons. Although, as we will see below, in most cases the calculations are plagued with strong interaction uncertainties making precise predictions impossible, it is of great interest to know at least the order of magnitude of the effects. This allows us to establish the existence or not of windows for the clean observation of new physics beyond the SM. This is particularly true in the case of mixing.

1 D^0-$\bar{D}^0$ mixing in the Standard Model

Mixing occurs because the two weak eigenstates D^0 and $\bar{D}^0$ are not the mass eigenstates. If we neglect CP violation, which as we will see below is a very good approximation for D mesons, the mass eigenstates are also CP eigenstates and can be written as

$$|D_1\rangle = \frac{1}{\sqrt{2}} \left(|D^0\rangle + |\bar{D}^0\rangle\right)$$
$$|D_2\rangle = \frac{1}{\sqrt{2}} \left(|D^0\rangle - |\bar{D}^0\rangle\right) \tag{1}$$

The probability that a D^0 meson produced at $t = 0$ decays as a $\bar{D}^0$ at time t is then given by

$$P(D^0 \rightarrow \bar{D}^0) = \frac{1}{4} e^{-\Gamma_1 t} \left\{1 - 2e^{-\frac{\Delta\Gamma}{2}t} \cos\Delta m t + e^{-\Delta\Gamma t}\right\} \tag{2}$$

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where $\Delta m = m_2 - m_1$ and $\Delta\Gamma = \Gamma_2 - \Gamma_1$ are the mass and lifetime differences in the mass eigenstates. These two quantities determine the ratio of "wrong" final state to "right" final state in decay modes in which the final state can only be reached by one of the neutral D meson flavors. This is the case in semileptonic decays where we can define

$$r_D = \frac{\Gamma(D^0 \to l^- X)}{\Gamma(D^0 \to l^+ X)} \tag{3}$$

This measurable quantity can be expressed in terms of Δm and $\Delta\Gamma$ by using (2) and the corresponding expression for the unmixed case. In the limit

$$\frac{\Delta m}{\Gamma}, \frac{\Delta\Gamma}{\Gamma} \ll 1 \tag{4}$$

it takes the simple form

$$r_D \approx \frac{1}{2}\left[\left(\frac{\Delta m}{\Gamma}\right)^2 + \left(\frac{\Delta\Gamma}{2\Gamma}\right)^2\right] \tag{5}$$

As we will see, (4) is a very good approximation.

In the SM r_D is expected to be very small. The question is how small. In this workshop the possibility of having 10^8 reconstructed D's in various experiments has been discussed [1]. It is expected that in some cases a sensitivity of 10^{-5} in r_D could be reached [2]. Several scenarios for new physics give contributions to r_D at this level. Therefore it is of great interest to establish at what level the SM contributes. It is not possible to compute r_D precisely, given the theoretical uncertainties arising from long distance dynamics. Unlike B^0-$\bar{B}^0$ mixing, where r_B is completely dominated by the short distance effects generated by the top quark, the inherently nonperturbative physics associated with these long distance effects (e.g. propagation of light quark intermediate states) is potentially large. In what follows we review the status of our knowledge of the short and long distance contributions to Δm. The lifetime difference $\Delta\Gamma$ is expected to be of the same order of magnitude as Δm. Given that we are interested in an order of magnitude estimate we will concentrate on Δm.

1.1 Δm_D: Short Distance

An effective $\Delta C = 2$ interaction is induced, at short distances, by one loop diagrams like the one in Fig. 1, the box diagrams. After the loop integration one obtains [3]

$$\mathcal{H}_{eff}^{\Delta C=2} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{8\pi \sin^2\theta_W} |V_{cs}^* V_{us}|^2 \frac{(m_s^2 - m_d^2)^2}{m_W^2 m_c^2} (\mathcal{O} + \mathcal{O}') \tag{6}$$

where in addition to the usual operator

$$\mathcal{O} = \bar{u}\gamma_\mu(1 - \gamma_5)c\bar{u}\gamma_\mu(1 - \gamma_5)c \tag{7}$$

one has to consider

$$\mathcal{O}' = \bar{u}(1 + \gamma_5)c\bar{u}(1 + \gamma_5)c \tag{8}$$

arising from the fact that the mass of the charm quark is not negligible. In (6) we
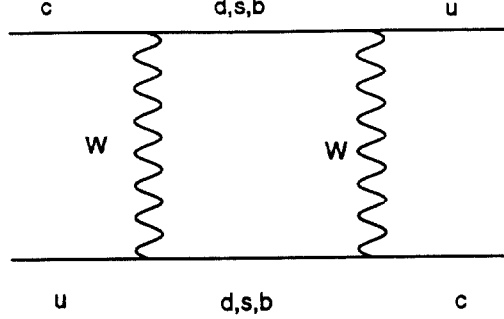
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Figure 1: One of the box diagrams that induce a $\Delta C = 2$ interaction.

neglect powers of m_q/m_W with $q = d, s$ and the b quark contribution that, although enhanced by a factor of $(m_b/m_W)^2$ is largely suppressed by the factor $|V_{ub}^* V_{cb}|^2$. The GIM mechanism produces the suppression factor $(m_s^2 - m_d^2)/m_W^2$: the effect vanishes in the $SU(3)$ limit. The additional suppression $(m_s^2 - m_d^2)/m_c^2$ comes from the fact that the external momentum, of the order of m_c, is communicated to the light quarks in the loop. Both factors explain why the box diagrams are so small for D mesons relative to the K and B mesons, where the GIM mechanism enters as m_c^2/m_W^2 and m_t^2/m_W^2 and external momenta can be neglected.

The mass difference generated by the box diagrams is

$$\Delta m = 2\langle D^0 | \mathcal{H}_{eff}^{\Delta C=2} | \bar{D}^0 \rangle \tag{9}$$

where the matrix elements of the operators $\mathcal{O}$ and $\mathcal{O}'$ can be parametrized as

$$\langle D^0 | \mathcal{O} | \bar{D}^0 \rangle = \frac{8}{3} m_D f_D^2 B_D \tag{10}$$

$$\langle D^0 | \mathcal{O}' | \bar{D}^0 \rangle = -\frac{5}{3} \left(\frac{m_D}{m_c} \right)^2 m_D f_D^2 B_D' \tag{11}$$

The vacuum insertion approximation, corresponding to the saturation of a sum over intermediate states by the vacuum state, gives $B_D = B_D' = 1$. Corrections to this simplified approach to the matrix elements are potentially large, but are not expected to change the order of magnitude of the effect. Therefore the box diagram contribution to the mass difference is

$$\Delta m_D^{s,d.} \approx 0.5 \times 10^{-17} \text{GeV} \left(\frac{m_s}{0.2\text{GeV}} \right)^4 \left(\frac{f_D}{f_\pi} \right)^2 \tag{12}$$

With the D^0 lifetime from [4] we have $\Gamma = (1.59 \pm 0.02) \times 10^{-12}$ GeV. Taking into account that the short distance contribution to $\Delta \Gamma$ is of the same order as (12), we use (5) to obtain the short distance contribution to the mixing parameter to be
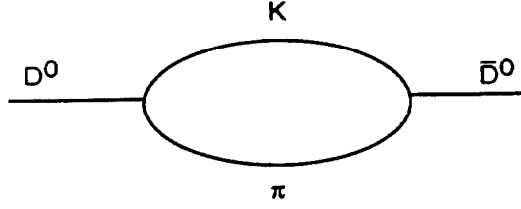
$$r_D^{s,d.} \approx 10^{-10} - 10^{-8} \tag{13}$$

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Figure 2: Two pseudoscalar intermediate state. There can be also KK and $\pi\pi$.

which is extremely small.

1.2 Δm: Long Distance

1.2.1 Dispersive Approach.

It has been argued that the fact that the main contributions to intermediate states in D meson mixing come from light quarks signals the presence of large long distance effects. They correspond to hadronic intermediate states propagating between the D mesons. It is, in principle, not possible to calculate these effects given their essentially nonperturbative character. However it is crucial to estimate their order of magnitude. In order to obtain it the authors of Ref. [5] make use of dispersive techniques. They consider sets of n-particle intermediate states related by $SU(3)$. In the $SU(3)$ limit the contribution from each of these sets must vanish. For instance, consider the two charged pseudoscalar intermediate states $K^-K^+, \pi^-\pi^+, K^-\pi^+, K^+\pi^-$. Their contribution to mixing comes from diagrams as the one in Fig. 2. Calculating the loop one typically obtains

$$\Sigma(p^2) = A(g)\left[\ln\left(-p^2\right) + \ldots\right] \tag{14}$$

where $A(g)$ depends on the form of the interaction and on the coupling g. The ellipses denote constant terms that also depend on the form of the vertex. However the logarithm gives an imaginary part that is related to the partial width of the on-shell intermediate state. That is, using

$$\ln\left(-p^2\right) = \ln p^2 + i\pi \tag{15}$$

the relation

$$Im\left[\Sigma(p^2)\right] = \Gamma/2 \tag{16}$$

fixes the coefficient of the logarithm. Keeping only this term and properly adding all the charged pseudoscalar states one obtains

$$\Delta m_D^{l.d.} \approx \frac{1}{2\pi}\ln\frac{m_D^2}{\mu^2}\left[\Gamma\left(D^0 \to K^-K^+\right) + \Gamma\left(D^0 \to \pi^-\pi^+\right)\right.$$
$$\left. -2\sqrt{\Gamma\left(D^0 \to K^-\pi^+\right)\Gamma\left(D^0 \to K^+\pi^-\right)}\right] \tag{17}$$

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where μ is a typical hadronic scale ($\simeq 1$ GeV). In order to get an estimate for the long distance effect we would need more information on the doubly Cabibbo suppressed mode $D^0 \to K^+\pi^-$. If we define

$$\frac{\Gamma\left(D^0 \to K^+\pi^-\right)}{\Gamma\left(D^0 \to K^-\pi^+\right)} = a \times \tan^4\theta_c, \tag{18}$$

then in the $SU(3)$ limit one would expect $a = 1$. However, a recent measurement by the CLEO collaboration gives [6]

$$a = 2.95 \pm 0.95 \pm 0.95, \tag{19}$$

signaling a possibly large breaking of $SU(3)$. Although the value of Δm_D must be proportional to the amount of $SU(3)$ breaking, the value of (19) does not mean the effect is necessarily large. Large $SU(3)$ breaking also occurs in the ratio [4]

$$\frac{\Gamma\left(D^0 \to K^+K^-\right)}{\Gamma\left(D^0 \to \pi^+\pi^-\right)} \simeq 3, \tag{20}$$

thus allowing for a partial cancellation of large $SU(3)$ breaking effects in (17). In the end the result can be expressed as

$$\frac{\Delta m_D^{l.d.}}{\Gamma} \simeq 8 \times 10^{-4} \left(1.4 - \sqrt{a}\right) \simeq -2.5 \times 10^{-4}, \tag{21}$$

where the last number corresponds to taking the central value in (19). However it can be seen that within the large error bars in (19) the effect is consistent with zero and more data is needed.

One could imagine computing, in the same fashion, contributions from other $SU(3)$ related sets of intermediate states: pseudoscalar-vector, vector-vector, three pseudoscalars, etc. All of these are proportional to the amount of $SU(3)$ breaking in the set. The relative signs of these contributions are unknown and although there could be cancellations one would expect the order of magnitude to stay the same.

1.2.2 Heavy Quark Effective Theory (HQET).

The applicability of the HQET ideas to D-$\bar{D}$ mixing rests on the assumption that the charm quark mass is much larger than the typical scale of the strong interactions. It was first pointed out in Ref. [7] that in this case there are no nonleptonic transitions to leading order in the effective theory since they would require a large momentum transferred from the heavy quark to the light degrees of freedom. This means that, in the effective low energy theory, mixing is a consequence of matching the full $\Delta C = 2$ theory at the scale m_c with the HQET and then running down to hadronic scales ($\ll m_c$). In other words, there are no new operators at low energy and the only "long distance" effects come from the renormalization group running below the matching scale m_c. As a consequence, Δm_D can be computed in the HQET using quark operators and restricting the nonperturbative physics only to their matrix elements, which in Ref. [6] are estimated using naive dimensional analysis.
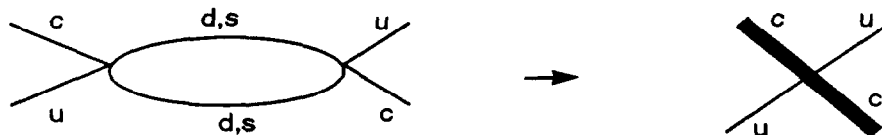
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Figure 3: Diagrams generating four-quark operators in HQET.

First let us consider the four-quark operators generated from the box diagrams by integrating out the W's. These and their matching diagrams in the effective theory are shown in Fig. 3. The contribution of these operators to the mass difference behaves like [7]

$$\Delta m_D^{(4)} \sim \frac{1}{16\pi^2} \frac{m_s^4}{m_c^2},\tag{22}$$

where the first factor comes from the loop and m_d is neglected. This is nothing but the HQET version of the box diagrams.

There will also be higher dimension operators. In principle they will be suppressed by additional powers of $1/m_c$. However, as we see below, they can give important contributions. For instance, six-quark operators are suppressed by one of such powers. We can think that they arise by "cutting" one of the light quark lines in the loop in Fig. 4 and then shrinking the connecting line leftover when going to the effective theory given that the momentum flowing through it is large ($\sim m_c$). As a consequence, we get rid of two powers of m_s and the contribution from six-quark operators goes like

$$\Delta m_D^{(6)} \sim \frac{1}{m_c} \frac{m_s^2}{m_c^2} \left(m_s f^2\right),\tag{23}$$

where the last factor comes from taking the hadronic matrix elements and f is the pseudo-goldstone boson decay constant.

Finally, eight quark operators are obtained by cutting the remaining light quark line and bridging the two four quark pieces with a gluon. The resulting contribution goes like

$$\Delta m_D^{(8)} \sim \frac{\alpha_s}{4\pi} \frac{1}{m_c^2} \frac{(m_s f^2)^2}{m_c^2},\tag{24}$$

As one can see from (24), this is the least GIM suppressed contribution. However it is suppressed by $1/m_c^2$ and most importantly by the factor $\alpha_s/4\pi$. Relative to the box diagram this is

$$\frac{\Delta m_D^{(8)}}{\Delta m_D^{(4)}} \simeq \frac{\alpha_s}{4\pi} \frac{(4\pi f)^4}{m_s^2 m_c^2} \simeq \frac{\alpha_s}{4\pi} \times 20\tag{25}$$

Therefore there is no enhancement due to these operators. In Ref. [7] it is argued that these contributions correspond to the intermediate states taken into account by the dispersive

approach. Thus the suppression factor $\alpha_s/4\pi$ in (24) suggests that there are cancellations among the different sets of states.

The six-quark operators give an enhancement of the order of

$$\frac{\Delta m_D^{(6)}}{\Delta m_D^{(4)}} \simeq \frac{(4\pi f)^2}{m_s m_c} \simeq 3 \tag{26}$$

A complete calculation in this approach, including QCD corrections to one loop, is performed in Ref. [8]. Their results can be summarized as

$$\Delta m_D^{(4)} \simeq (0.5 - 0.9) \times 10^{-17} \text{GeV} \left(\frac{m_s}{0.2\text{GeV}}\right)^4$$

$$\Delta m_D^{(6)} \simeq (0.7 - 2.0) \times 10^{-17} \text{GeV} \left(\frac{m_s}{0.2\text{GeV}}\right)^3$$

$$\Delta m_D^{(8)} \simeq (0.1 - 0.6) \times 10^{-17} \text{GeV} \left(\frac{m_s}{0.2\text{GeV}}\right)^2$$

In sum, the HQET approach to Δm_D predicts

$$\frac{\Delta m_D}{\Gamma} \simeq (1 - 2)10^{-5} \tag{27}$$

The uncertainty in (27) is mostly due to the uncertainty in the relative signs of the various contributions. However is clear that HQET predicts no large enhancements with respect to the box diagram, which implies a mixing parameter of the order of

$$r_D \approx 10^{-10} - 10^{-9}. \tag{28}$$

In conclusion, with the current data on DCSD there seems to be no large disagreement between the dispersive approach of Ref. [5] and the HQET estimate of the mixing parameter for D mesons [7, 8]. A conservative upper limit can then be established for the SM contribution to D^0-$\bar{D}^0$ mixing to be

$$r_D^{SM} < 10^{-8} \tag{29}$$

2 CP Violation

In order for CP violation to occur there must be at least two amplitudes interfering with non-zero relative phases. There are two mechanisms that can produce this interference. In the first case the two amplitudes correspond to a D^0 decaying as a D^0 at time t and a D^0 decaying, after mixing, as a $\bar{D}^0$ at time t, both to the same final state f. This is called indirect CP violation and is theoretically clean. That is, the hadronic uncertainties cancel in the asymmetry given that they are the same for both amplitudes. However, as we have seen in the previous section, the mixing amplitude is extremely small in the SM and therefore the induced CP violation is negligible.

More generally, CP violation can occur directly in the decay amplitude. Let us assume two amplitudes contribute to a given D decay mode. Then

$$A_f = A_1 e^{i\delta_1} + A_2 e^{i\delta_2} \tag{30}$$

where A_1 and A_2 are the two amplitudes after factoring out the strong interaction phases δ_1 and δ_2. When the CP conjugate is taken the weak phases included in $A_{1,2}$ change but the strong phases stay the same:

$$\bar{A}_f = A_1^* e^{i\delta_1} + A_2^* e^{i\delta_2} \tag{31}$$

The CP asymmetry is then

$$a_{CP} = \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2} = \frac{2 Im \left[A_1^* A_2 \right] \sin\left(\delta_1 - \delta_2 \right)}{|A_1|^2 + |A_2|^2 + 2 Re \left[A_1^* A_2 \right] \cos\left(\delta_1 - \delta_2 \right)} \tag{32}$$

From (32) we see that in order to have a nonzero asymmetry the two amplitudes must have different weak as well as strong phases. The predictions for a_{CP} are then plagued with hadronic uncertainties coming from the amplitudes and the final state interaction phases.

The interesting question is what is the typical size of the effect in the SM. Before going into the more detailed analysis let us remember that any CP violating effect in the SM must be proportional to

$$J = Im \left[V_{ij} V_{kl} V_{ik}^* V_{jl}^* \right] \tag{33}$$

for any choice of $i \neq l$ and $j \neq k$. With the current values of the CKM phases and taking for the CP violating phase $\sin \delta = 1$ we know that $J \leq 10^{-4}$. From (32) we can see that CP asymmetries are larger the more suppressed is the mode . For instance, for Cabibbo suppressed decays we have an enhancement of $\sin^{-2}\left(\theta_c \right)$ and then an order of magnitude estimate for the asymmetry is

$$a_{CP} \sim 10^{-3} \tag{34}$$

In D decays all tree level diagrams contributing to a given final state have the same CKM matrix element combination. They will interfere only with the one loop diagrams called penguins. Cabibbo favored D modes do not have penguins and then we are left with Cabibbo suppressed decays, for which the asymmetry is estimated in (34). However the fact that one of the amplitudes is likely to be much smaller, the penguin in this case, largely reduces the size of the asymmetry. The relative size of the penguin to the tree level diagrams is not a settled issue but one should consider (34) to be on the rather optimistic side unless there is a large enhancement from strong interaction dynamics, in the same fashion as in the $\Delta I = 1/2$ rule. This possibility is raised in Ref. [9]

On the other hand, in D_s decays it is possible to have two tree level amplitudes with different weak phases. For instance in $D_s \to K\pi$ the spectator and annihilation diagrams are proportional to $V_{cd}^* V_{ud}$ and $V_{cs}^* V_{us}$ respectively. Therefore, if the annihilation diagram is not suppressed relative to the spectator, asymmetries of the order of (34) are expected.

As it was mentioned above, the calculation of the asymmetries involves the knowledge of hadronic matrix elements and strong interaction phases. This is done, for instance, in Refs. [10] and [11]. In the first case, the relative strong phases are provided by the quark diagrams and final state interactions are neglected.

In the work of Ref. [11], large final state interaction phases are provided by nearby resonances. This tends to give larger asymmetries. The typical result in this case is a few $\times 10^{-3}$. For instance, for the decay $D^+ \to \bar{K}^* K^+$ $a_{CP} = 2.8 \times 10^{-3}$. In D_s decays the most interesting mode is $K^* \eta'$ with $a_{CP} = -8.1 \times 10^{-3}$.

In any event, all calculations of direct CP violation asymmetries are very uncertain. The SM can give at most an effect of the order of 10^{-3} but more precise predictions are not possible with our current imprecise knowledge of hadronic physics.

Finally, we mention the possibility of kinematic CP violation signals. For instance, in decays to two vector mesons $D(p) \to V_1(k)V_2(q)$ [12, 13], it is possible to construct CP odd correlations of the two polarizations and one of the momenta. A triple product correlation $\langle k.\epsilon_1 \times \epsilon_2 \rangle$ is T odd. However a non-vanishing value of this quantity is not necessarily a signal of CP violation: the effect could be entirely due to strong interaction phases. In order to have a truly CP odd correlation one has to compare with the CP conjugate state: the sum of

$$N_f = \frac{N(k.\epsilon_1 \times \epsilon_2 > 0) - N(k.\epsilon_1 \times \epsilon_2 < 0)}{N_{total}} \tag{35}$$

and the corresponding quantity for the CP conjugate state, $N_{\bar{f}}$, should vanish if CP is conserved. Similar correlations but for semileptonic decays are discussed in [14]. Another type of kinematic signal can be obtained in neutral three body decays like $D^0 \to M^+ M^- N^0$ [15]. In general the partial decay rate of a given neutral D flavor need not be symmetric in the energies E_+ and E_-. However when adding all reconstructed neutral D's from the final state without identifying the D flavor, the Dalitz plot must be symmetric in E_+, E_- unless CP is violated. That is, given the expression

$$\Gamma\left[(D^0 + \bar{D}^0) \to M^+ M^- N^0\right] = a + b(E_+ - E-) \tag{36}$$

an nonzero value of b signals a net energy asymmetry and therefore CP violation.

In all cases, the kinematic asymmetries are also plagued with hadronic uncertainties as in the case of partial rate asymmetries in charged D decays. However it is important that they are taken into account given that in some cases they might be easier to observe.

To summarize, the SM predicts that CP violation in charm decays proceeds via the direct mechanism given the small value of r_D. Asymmetries are expected to be at most of order 10^{-3} in modes with branching fractions of 10^{-3}. This implies the need of at least 10^8 reconstructed D's in order to observe a 3σ effect.

3 Conclusions

We have seen that the SM predicts extremely small values for the mixing parameter r_D. The effect, even after including possible long distance enhancements, seems to be in the range $10^{-10} - 10^{-8}$. These effects had been previously overestimated in [16] giving therefore the impression that any observation of D^0-$\bar{D}^0$ mixing would be contaminated by long distance dynamics. However this is not the case. An observation of D mixing at the level of $10^{-4} - 10^{-5}$, which is going to be probed at high sensitivity experiments, would be a signal of new physics [17].

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On the other hand, SM CP violation might be marginally observable in some cases. New physics signals could then be mixed with these although there are models where sizeable asymmetries occur in Cabibbo favored modes, giving a clear signal over the SM background [17].

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